LARGE SCALE STRUCTURE EFFECTS ON THE GRAVITATIONAL LENS IMAGE POSITIONS AND TIME DELAY

Uroš Seljak[1]
Department of Physics, MIT, Cambridge, MA 02139 USA

ABSTRACT

We compute the fluctuations in gravitational lens image positions and time delay caused by large scale structure correlations. We show that these fluctuations can be expressed as a simple integral over the density power spectrum. Using the *COBE* normalization we find that positions of objects at cosmological distances are expected to deviate from their true positions by a few arcminutes. These deflections are not directly observable. The positions of the images relative to one another fluctuate by a few percent of the relative separation, implying that one does not expect multiple images to be produced by large scale structures. Nevertheless, the fluctuations are larger than the observational errors on the positions and affect reconstructions of the lens potential. The time delay fluctuations have a geometrical and a gravitational contribution. Both are much larger than the expected time delay from the primary lens, but partially cancel each other. We find that large scale structure weakly affects the time delay and time delay measurements can be used as a probe of the distance scale in the universe.

Subject headings: gravitational lenses — cosmology: large-scale structure of the universe

1. Introduction

[1]Also Department of Physics, University of Ljubljana, Jadranska 19, 61000 Ljubljana, Slovenia

The possibility of studying the physical parameters of the distant universe using gravitational lenses (GL) was first suggested in the 1960s. In particular, Refsdal (1964, 1966) suggested that one could determine the masses of galaxies and the Hubble constant using the observed image properties, most notably their positions, magnifications and time delays between images of the same source. The latter became especially interesting after the time delay in the system 0957+561 was measured (e.g. Vanderriest et al. 1989; Lehár et al. 1992) and a value of H_0 derived (Rhee 1991; Roberts et al. 1991). Alcock & Anderson (1985, 1986), Watanabe, Sasaki & Tomita (1992) and Sasaki (1993) criticized the method, arguing that large scale structure (LSS) might significantly affect the time delay. Unfortunately, their arguments were only qualitative and could not give realistic predictions of the amplitude of fluctuations. For example, Falco, Gorenstein & Shapiro (1991) found that the effect Alcock & Anderson discussed can change the derived value of H_0 by only a few percent. In light of this many workers in the field have taken an optimistic view and assumed that the derived value of H_0 gives at least an upper limit to the actual value (e.g. Borgeest & Refsdal 1984). These arguments are based on the fact that mass density is always positive and therefore always focuses the rays. However, this is only correct in Newtonian gravity and becomes invalid in cosmological applications, where underdensities such as voids give an effective negative mass density (e.g. Nityananda & Ostriker 1984). In general, the question whether LSS could significantly affect the measured properties of the lens has remained largely unanswered.

In this paper we present a calculation of position and time delay dispersions in a GL system. The calculation is done using linearized general relativity for some realistic *COBE* normalized cosmological models. In §2 we present the method, based on the geodesic equation for a perturbed Robertson-Walker metric. A similar approach has been used by Linder (1990) and Cayón, Martínez-González & Sanz (1993a, 1993b) to study the GL effects on the cosmic microwave background and by Kaiser (1992) to derive the ellipticity correlation function of distant galaxies. An alternative method, based on optical scalars, has been developed by Gunn (1967) and applied to the ellipticity and magnification correlation function calculations by Babul & Lee (1991), Blandford et al. (1991) and Miralda-Escudé (1991). In §3 we apply the method to compute the fluctuations in the image position relative to the true position and relative to another image position. In §4 we present the calculation of fluctuations in time delay. In §5 we present the conclusions and comment on their agreement with previous work on this subject.

2. Formalism

Our framework is a perturbed flat Robertson-Walker model with small-amplitude scalar metric fluctuations. In the longitudinal gauge (Bardeen 1980; Mukhanov, Feldman and Brandenberger 1992) one can write the line element as

$$ds^2 = a^2(\tau)\Big[-(1+2\phi)d\tau^2 + (1-2\phi)d\boldsymbol{x}\cdot d\boldsymbol{x}\Big]\,. \tag{1}$$

We assume that $|\phi| \ll 1$ and neglect all the terms of order $O(\phi^2)$ and higher. This is a good approximation almost everywhere in the universe, except near black holes. We adopt units such that $c = 1$. In the line element above we neglected the contributions from vector and tensor modes and the gravitational effects from anisotropic stresses. These approximations are valid nearly always, especially in the regime of interest for us, which is the matter dominated era with fluctuation wavelengths small compared to the Hubble distance. In this case ϕ can be interpreted as the Newtonian potential and, neglecting the contributions from wavelengths larger than the Hubble distance, it obeys the cosmological Poisson equation

$$\nabla^2\phi = 4\pi G a^2(\rho - \bar{\rho}), \tag{2}$$

where ρ is the local density and $\bar{\rho}$ the mean density in the universe (e.g. Bertschinger 1993). We denote the time dependence of the potential with $F(\tau)$, which is independent of the scale in linear perturbation theory, assuming the dominant matter component has negligible Jeans length. For the zero curvature model with a cosmological constant, $\Omega_m + \Omega_\lambda = 1$, one has (Heath 1977)

$$F(a) = \frac{\sqrt{\Omega_m + \Omega_\lambda a^3}}{a^{5/2}}\frac{\int_0^a X^{3/2}da}{\int_0^1 X^{3/2}da}, \tag{3}$$

where $X = a/(\Omega_m + \Omega_\lambda a^3)$ and $H_0\tau = \int_0^a da/(\Omega_m a + \Omega_\lambda a^4)^{1/2}$; we normalize $a(\tau) = 1/(1+z)$ to $a = 1$ today. H_0 is the Hubble constant and Ω_m and Ω_λ are the matter and vacuum densities, respectively, in units of the critical density. The simplest model has $\Omega_m = 1$, for which $F(\tau) = 1$ and $a(\tau) = (H_0\tau/2)^2$.

Suppose a photon is emitted from a distant source toward the observer (see Figure 1). If there are no perturbations present the photon will travel along a null geodesic in the radial direction [2] , $\boldsymbol{x}(r) = r\hat{\boldsymbol{r}}$ and $r(\tau) = \tau_0 - \tau$, where τ_0 is the conformal time today. Adding a perturbation changes the photon trajectory into $\boldsymbol{x}(r) = r\hat{\boldsymbol{r}} + \boldsymbol{x}_\perp(r)$, where $\boldsymbol{x}_\perp(r)$ is the excursion of the photon in the direction orthogonal to $\hat{\boldsymbol{r}}$. The 3-tangent to the geodesic curve is given by $\boldsymbol{n} = d\boldsymbol{x}/dl$, where $dl = (d\boldsymbol{x}\cdot d\boldsymbol{x})^{1/2}$ is the comoving path length along the geodesic. Null geodesics obey the relation $d\tau = (1-2\phi)dl$. Even when

[2] Here and throughout the paper all 3-vectors are defined in the unperturbed comoving coordinates of 3-space, which is a hypersurface of constant τ. The geometry of this 3-space is a simple Euclidean geometry.

metric perturbations are present, we can continue to parametrize the geodesic with the unperturbed radial coordinate r. The relation between r and l is given by $dr/dl = \boldsymbol{n} \cdot \hat{\boldsymbol{r}}$. The linearized space part of the photon geodesic equation derived from the metric in equation (1) gives the rate of change in photon direction,

$$\frac{d\boldsymbol{n}}{dl} = 2\boldsymbol{n} \times (\boldsymbol{n} \times \nabla\phi) \tag{4}$$

(e.g. Weinberg 1972 eq. 9.2.6-7; note that there is a factor of 2 missing in eq. 9.2.7). For weak gravitational fields ($|\phi| \ll 1$), ϕ can be viewed as providing a force deflecting the photons and affecting their travel time while they propagate through unperturbed space-time. We shall adopt this quasi-Newtonian interpretation throughout this paper.

Because $\boldsymbol{n}$ is normalized, it is sufficient to consider only its two components orthogonal to $\hat{\boldsymbol{r}}$. We define the two-dimensional photon direction angle $\boldsymbol{\gamma}(r) = [\gamma_1(r), \gamma_2(r)]$ relative to the unperturbed photon direction $-\hat{\boldsymbol{r}}$ (taken as the third direction) as $\boldsymbol{n} = (\gamma_1, \gamma_2, -1)$. We assume small deflection angles (this will be justified in §3).

The evolution of $\boldsymbol{\gamma}(r)$ is given by

$$\boldsymbol{\gamma}(r) = \boldsymbol{\gamma}(r_{OS}) + \boldsymbol{\alpha}_\perp(r, r_{OS}), \tag{5}$$

where $\boldsymbol{\gamma}(r_{OS})$ is the initial photon direction (here r_{OS} is the comoving distance between the observer and the source) and $\boldsymbol{\alpha}_\perp(r, r_{OS}) = [\alpha_{\perp,1}(r, r_{OS}), \alpha_{\perp,2}(r, r_{OS})]$ is a two-dimensional deflection angle produced by the potential along the geodesic between the source and the point under consideration at r. From equation (4) follows

$$\boldsymbol{\alpha}_\perp(r, r_{OS}) = \int_r^{r_{OS}} \boldsymbol{g}_\perp(r) dr. \tag{6}$$

We introduced $\boldsymbol{g}_\perp(r)$ defined as

$$\boldsymbol{g}_\perp(r) = -2\nabla_\perp\phi[\boldsymbol{x}(r), \tau(r)] = -2[\nabla - \hat{\boldsymbol{r}}(\nabla \cdot \hat{\boldsymbol{r}})]\phi[\boldsymbol{x}(r), \tau(r)]. \tag{7}$$

The photon excursion in the plane perpendicular to $\hat{\boldsymbol{r}}$ is given by

$$\boldsymbol{x}_\perp(r) = \int_r^{r_{OS}} \boldsymbol{\gamma}(r) dr. \tag{8}$$

The initial photon direction $\boldsymbol{\gamma}(r_{OS})$ must be chosen so that $\boldsymbol{x}_\perp(0) = 0$, i.e., the ray must pass through the observer's position. This gives the lens equation

$$\int_0^{r_{OS}} \boldsymbol{\gamma}(r) dr = 0$$

or

$$r_{OS}\boldsymbol{\gamma}(r_{OS}) = -\int_0^{r_{OS}} \boldsymbol{\alpha}_\perp(r, r_{OS})dr. \tag{9}$$

This lens equation is valid for an arbitrary mass distribution between the source and the observer. It cannot be solved explicitly in general, because $\boldsymbol{\alpha}_\perp(r, r_{OS})$ depends on the unknown initial photon direction $\boldsymbol{\gamma}(r_{OS})$. Instead, one has to solve an integral equation using, for example, the ray-shooting method.

We may also ask what is the photon direction at the observer's position so that $\boldsymbol{x}_\perp(r_{OS}) = 0$. This is given by the image position angles $\boldsymbol{\gamma}(0)$. For the special case of a single thin lens one has

$$\boldsymbol{\alpha}_\perp(r, r_{OS}) = \begin{cases} 0, & r > r_{OL} \\ \widehat{\boldsymbol{\alpha}}, & r \leq r_{OL}, \end{cases} \tag{10}$$

where $\widehat{\boldsymbol{\alpha}}$ is the bending angle in the lens plane and r_{OL} is the comoving distance between the observer and the lens. Equation (9) then gives

$$r_{OS}\boldsymbol{\gamma}(0) = -r_{LS}\widehat{\boldsymbol{\alpha}}, \tag{11}$$

which is the usual lens equation in the thin lens approximation (e.g. Schneider, Ehlers & Falco 1992; Blandford & Kochanek 1987). Here r_{LS} is the comoving distance between the observer and the source and $r_{OS} = r_{OL} + r_{LS}$.

The distances can be calculated for a given cosmological model from the source redshift z_s and lens redshift z_l (assuming that the two redshifts can be measured). For example, for $\Omega_m = 1$ one has $r_{OL} = 2H_0^{-1}[1-(1+z_l)^{-1/2}]$ and $r_{LS} = 2H_0^{-1}[(1+z_l)^{-1/2} - (1+z_s)^{-1/2}]$. We use the unperturbed comoving distance-redshift relation, because the deviations from it are $O(\phi)$ and can be neglected to the first order. Note that we expressed the lens equation using comoving distances. One can reexpress it, if one wishes, with angular diameter distances using the relation $d_{AS} = r_{AS}/(1 + z_S)$, where z_S is the redshift of the source and d_{AS} and r_{AS} are, respectively, the angular diameter distance and comoving distance between the point A and the source.

Similarly, the net time delay along the photon path relative to the unperturbed path is given by

$$\Delta t = \Delta\tau(a = 1) = \int_0^{r_{OS}} \left[\frac{1-2\phi(r)}{\boldsymbol{n}\cdot\widehat{\boldsymbol{r}}} - 1\right]dr \approx$$
$$\int_0^{r_{OS}} \left[\frac{1}{2}\boldsymbol{\gamma}^2(r) - 2\phi[\boldsymbol{x}(r), \tau(r)]\right]dr, \tag{12}$$

where $\boldsymbol{\gamma}^2(r) = \gamma_1^2(r) + \gamma_2^2(r)$ and we used the small angle approximation; the unperturbed null geodesic equation may be used for $\tau(r) = \tau_0 - r$. The first and second term in equation

(12) are the geometrical and gravitational (also called potential or Shapiro) time delay contribution, respectively. Again, for the particular case of the thin lens approximation this reproduces the usual time delay equation, as shown in §4.

Although we have derived our equations using the geodesic equations of a perturbed Robertson-Walker metric, the final expressions agree with the usual thin lens equations, when we interpret ϕ as the Newtonian potential, obeying equation 2. In addition, one does not have to assume that density perturbations are small, as long as $|\phi| \ll 1$. These are advantages of the longitudinal gauge compared to other gauge choices (e.g. synchronous gauge). The particular strength of the approach presented here is that it remains valid in cosmological applications and can give the deflection angle and time delay due to any matter distribution between the source and observer. Use of comoving coordinates and conformal time greatly simplifies the GL equations. In particular, all the equations above use a simple Euclidean spatial geometry and there is no need to use angular distances or worry about expansion factors. The general expressions above would be more cumbersome when expressed with angular diameter distances.

We are interested in LSS effects on the image properties of lenses. Because of the statistical nature of cosmological theories one can only predict the ensemble averages of a given quantity, such as its mean and variance. All the GL effects are given through the gravitational potential $\phi[\boldsymbol{x}, \tau(r)]$. Its statistical properties can best be described with the Fourier transformed potential $\phi(\boldsymbol{k})$,

$$\phi[\boldsymbol{x}, \tau(r)] = \int d^3k \phi(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} F[\tau(r)]. \tag{13}$$

The ensemble mean and variance of the Fourier transform of the potential are $\langle\phi(\boldsymbol{k})\rangle = 0$ and $\langle\phi(\boldsymbol{k})\phi^*(\boldsymbol{k'})\rangle = P_\phi(k)\delta^3(\boldsymbol{k} - \boldsymbol{k'})$, where $P_\phi(k)$ is the power spectrum of the potential (see e.g. Bertschinger 1992). We will use the power spectrum of the potential in this paper because it leads to the simplest expressions, but note that one can easily reexpress the results with the density power spectrum using equation 2.

A particularly convenient normalization of $P_\phi(k)$ is given by the cosmic microwave background anisotropy measured by *COBE* - the quadrupole $Q_2 = (\Delta T/T)_2 = (6 \pm 1) \times 10^{-6}$ (Smoot et al. 1992; Seljak & Bertschinger 1993). On the large scales probed by *COBE* the dominant contribution to $\Delta T/T$ is given by the Sachs-Wolfe (1967) effect, induced by the same metric fluctuations that cause the fluctuations in time delay and image positions. Assuming no tensor mode contribution to CMB anisotropies and isentropic (adiabatic) fluctuations we can express the quadrupole in terms of the power spectrum of the potential

(Bond & Efsthathiou 1987) as

$$Q_2^2 = \frac{20\pi K_2^2}{9}\int_0^\infty k^2 P_\phi(k) j_2^2(2k/H_0)dk. \tag{14}$$

Here $j_2(x)$ is the spherical Bessel function of order 2 and $K_2^2 \geq 1$ is the amplification coefficient due to the time dependent potential (Kofman & Starobinsky 1985). If $\Omega_m = 1$ the potential is time independent and $K_2^2 = 1$. For the scale-invariant Peebles-Harrison-Zel'dovich spectrum $P_\phi(k) = Ak^{-3}$ one gets

$$Q_2^2 = \frac{5\pi K_2^2 A}{27}. \tag{15}$$

3. Fluctuations in Angular Position

The first question we will ask is what is the rms fluctuation in the photon direction at the observer's position relative to the unperturbed direction. This is defined as

$$\sigma_\gamma = \frac{1}{\sqrt{2}}\langle \boldsymbol{\alpha}_\perp(0, r_{OS}) \cdot \boldsymbol{\alpha}_\perp(0, r_{OS})\rangle^{1/2}, \tag{16}$$

where $\boldsymbol{\alpha}_\perp(0, r_{OS})$ is the total deflection angle accumulated between the source and the observer, as defined in equation (6). In order to compute the variance we will make an additional assumption that the statistical properties of the potential sampled by the perturbed photon geodesic are approximately equal to those along the unperturbed one,

$$\langle \phi[\boldsymbol{x}, \tau(r)]\phi[\boldsymbol{x}, \tau(r)]\rangle \approx \langle \phi[r\hat{\boldsymbol{r}}, \tau(r)]\phi[r\hat{\boldsymbol{r}}, \tau(r)]\rangle. \tag{17}$$

This assumption will be discussed later.

We want to compute statistical properties of the transverse gradient of the potential field ϕ integrated along line of sight in the direction $\boldsymbol{\gamma}$ and weighted with a function $q(r)$,

$$\boldsymbol{p}(\boldsymbol{\gamma}) = -2\int_0^{r_0} [\boldsymbol{\nabla}_\perp \phi(r, \boldsymbol{\gamma})]q(r)dr. \tag{18}$$

We will use the Fourier space analog of Limber's equation (e.g. Kaiser 1992) to calculate the correlation function from the power spectrum,

$$C(\gamma = |\boldsymbol{\gamma}_1 - \boldsymbol{\gamma}_2|) = \langle \boldsymbol{p}(\boldsymbol{\gamma}_1) \cdot \boldsymbol{p}(\boldsymbol{\gamma}_2)\rangle = 8\pi^2 \int_0^\infty k^3 dk \int_0^{r_0} P_\phi[k, \tau(r)]q^2(r)J_0(kr\gamma)dr, \tag{19}$$

where $J_0(x)$ is the Bessel function of order 0. Equation 19 assumes that the scales contributing to the correlation function are much smaller than the photon pathlength r_0. As shown below, this is usually satisfied in most models of LSS for sources at cosmological distances. No assumption on the power spectrum has been made and equation 19 can be used both in linear and in non-linear regime. To compute the variance σ_γ we use the above expression with $\gamma = 0$, $q(r) = 1$ and $r_0 = r_{OS}$. Assuming linear evolution and $\Omega_m = 1$ we obtain

$$\sigma_\gamma = \left(8\pi^2 r_{OS} \int_0^\infty P_\phi(k) k^3 dk\right)^{1/2}. \tag{20}$$

The rms fluctuation in photon direction at the distance r_{OS} from the source, σ_γ, is related to the rms angular fluctuation of the true source position relative to the observed position. To compute this we have to back-propagate all the photons with a fixed final direction and ask what are their angular excursions in the source plane,

$$\sigma_\theta = \frac{1}{\sqrt{2}}\left\langle\left[\frac{1}{r_{OS}}\int_0^{r_{OS}} \boldsymbol{\alpha}_\perp(0,r)dr\right]\cdot\left[\frac{1}{r_{OS}}\int_0^{r_{OS}} \boldsymbol{\alpha}_\perp(0,r)dr\right]\right\rangle^{1/2}. \tag{21}$$

Integrating by parts the two terms in equation (21) we find that σ_θ is given by a similar expression as σ_γ with $q(r) = (1 - r/r_{OS})$. Using equation 20 gives

$$\sigma_\theta = \frac{\sigma_\gamma}{\sqrt{3}} = \left(\frac{8\pi^2 r_{OS}}{3}\int_0^\infty P_\phi(k) k^3 dk\right)^{1/2}. \tag{22}$$

To get some intuition about the scaling of the amplitude with the parameters we will present an estimate of the fluctuations for a particularly simple power spectrum approximating inflationary models with a physical transfer function:

$$P_\phi(k) = \begin{cases} Ak^{-3} & , k < k_0 \\ Ak^{-7}k_0^4 & , k > k_0 \end{cases} \tag{23}$$

The spectral indices have been chosen to agree with the cold dark matter model in the limits of small and large k. Applying this power spectrum to equations (20) and (22) we find

$$\sigma_\theta = \frac{\sigma_\gamma}{\sqrt{3}} \approx 7Q_2(k_0 r_{OS})^{1/2}. \tag{24}$$

This result has a simple physical interpretation. For power spectra like in equation (23) the dominant contribution to gravitational deflection of light comes from the scales near the turnover position k_0^{-1}. A photon travelling through a coherent structure of size k_0^{-1} will be deflected by $\delta\gamma \approx 2\phi \approx 6Q_2$, where the last relation assumes ϕ is scale invariant for $k < k_0$ and is therefore fixed by the Sachs-Wolfe effect on the Hubble distance scale. Each

region of size k_0^{-1} makes an independent contribution to the deflection. Since the individual contributions are random, the photon exhibits a random walk with $\sigma_\gamma \approx N^{1/2}\delta\gamma$, where $N = k_0 r_{OS}$. Numerical factors aside this agrees with equation (24). A reasonable value for the turnover position in the power spectrum is given by $k_0^{-1} = 10$ Mpc. Taking $r_{OS} = 1$ Gpc as a typical source distance we find $\sigma_\theta \approx 3 \times 10^{-4}(k_0 r_{OS}/100)^{1/2} \approx 1'(k_0 r_{OS}/100)^{1/2}$. This is small compared to 1, which justifies the small deflection angle assumption. It also verifies that the pathlengths are not significantly lengthened by the perturbations.

However, we conclude that the fluctuations in the angular position σ_θ are of the order of arcminutes (see also Linder 1990), which means that the true positions of distant objects in the universe, such as quasars, differ from the measured positions on average (rms) by this amount. These fluctuations arise already because of the linear structures (voids and superclusters) and are present even when there are no nonlinear objects (like galaxies and clusters) intersecting the photon trajectories. The fluctuations are much larger than a typical image separation in a lens system, which is of the order of a few arcseconds. The large total deflections are not directly observable in a single lens system, because only the relative positions of images can be measured. Figure 2 shows photon propagation in a typical two image GL system. Despite the fact that the deflection of any single photon ray can be large, the lens equation will still give the same solution as in the unperturbed case, provided that LSS deflects the two photons approximately by the same amount. This will be examined below. Although the deflection of a photon ray relative to the unperturbed direction is not directly observable from the positions, one might worry that it could produce significant time delay fluctuations. We will address this question in §4.

Let us now calculate LSS effects on the relative image positions, by calculating $\sigma_{\Delta\gamma}$ and $\sigma_{\Delta\theta}$, the dispersion in the relative direction and in the relative angular position separation between two image rays. We will denote the two rays with A and B, separated in direction at the observer's position by an angle $\Delta\gamma_0 = \gamma_1^A(0) - \gamma_1^B(0)$, where the orientation of the coordinate system was chosen so that the image separation vector lies in the direction $\boldsymbol{e_1}$ (so that $\gamma_2^A(0) = \gamma_2^B(0)$). We divide the potential into a stochastic part, which describes the LSS, and a non-stochastic part describing the primary lensing object. We assume there are no correlations between these two parts. Taking the expectation value of equation (9) the stochastic contributions average to 0 and we obtain the usual lens equation in the thin lens approximation.

The difference between the two direction vectors caused by LSS between the lens and the observer is given by

$$\Delta\boldsymbol{\gamma}(r_{OL}) = \Delta\gamma_0\boldsymbol{e_1} + \boldsymbol{\alpha}_\perp{}^A(0, r_{OL}) - \boldsymbol{\alpha}_\perp{}^B(0, r_{OL}), \tag{25}$$

where $\boldsymbol{\alpha}_\perp{}^A(0, r_{OL})$ and $\boldsymbol{\alpha}_\perp{}^B(0, r_{OL})$ are the LSS caused deflections for the rays A and B,

respectively. We excluded the non-stochastic deflection from the primary lens itself. For a fixed photon separation angle at the observer's position $\Delta\gamma_0$, the rms fluctuation in the angle between the two rays at the lens position is given by

$$\sigma_{\Delta\gamma} = \frac{1}{\sqrt{2}}\left\langle\left[\boldsymbol{\alpha}_\perp{}^A(0, r_{OL}) - \boldsymbol{\alpha}_\perp{}^B(0, r_{OL})\right]^2\right\rangle^{1/2} = \left[C(\Delta\gamma_0) - C(0)\right]^{1/2}, \tag{26}$$

where $C(\gamma)$ is given by equation (19) with $q(r) = 1$ and $r_0 = r_{OL}$. Assuming in addition that the scales contributing to the fluctuations are much larger than the typical separation between the two rays (one can always verify this assumption by evaluating $\sigma_{\Delta\gamma}$ from equation 19), we obtain

$$\sigma_{\Delta\gamma} = 2\pi\Delta\gamma_0\left[\frac{r_{OL}^3}{3}\int_0^\infty P_\phi(k)k^5 dk\right]^{1/2}. \tag{27}$$

Replacing $\Delta\gamma_0$ with $(r_{OL}/r_{LS})\Delta\gamma_0$ and r_{OL} with r_{LS} in equation 27 gives the rms fluctuation between the two ray directions accumulated between the source and the lens. Summing the two contributions in quadrature gives the rms fluctuation accumulated between the source and the observer, neglecting the correlations between paths on either side of the lens. Similarly, the dispersion of the angular position in the lens plane is given by setting $q(r) = (1 - r/r_{OL})$, which gives for the contribution between the observer and lens,

$$\sigma_{\Delta\theta} = \frac{\sigma_{\Delta\gamma}}{\sqrt{10}} = 2\pi\Delta\gamma_0\left(\frac{r_{OL}^3}{30}\int_0^\infty P_\phi(k)k^5 dk\right)^{1/2}. \tag{28}$$

For the simple power spectrum of equation (23) we find

$$\sigma_{\Delta\gamma} = \sqrt{10}\sigma_{\Delta\theta} \approx 4Q_2(k_0 r_{OL})^{3/2}\Delta\gamma_0 \approx 0.025(k_0 r_{OL}/100)^{3/2}\Delta\gamma_0. \tag{29}$$

Again, there is a simple physical explanation of this result. Two photons separated by an angle $\Delta\gamma_0$ sample different potentials, $\delta\phi \approx (\nabla_\perp\phi)r\Delta\gamma_0 \approx k_0\phi r\Delta\gamma_0$, including only the peak power contribution around k_0. The separation $r\gamma_0$ is largest at the lens, but only falls to one-half for distances half and three-halves as far, so it is a reasonable approximation to fix r to r_{OL}. A coherent structure of size k_0^{-1} leads to an angular difference of $\delta\Delta\gamma \approx 2\delta\phi$ and there are $N = k_0 r_{OL}$ random and independent contributions. The total angular difference is just an incoherent sum of individual contributions, $\sigma_{\Delta\gamma} \approx N^{1/2}\delta\Delta\gamma$, which, numerical factors aside, reproduces equation (29).

A more direct way to estimate the amplitude of image position fluctuation is to use the observations of correlated distortions of distant galaxy images. This can be described by the ellipticity correlation function $C_{pp}(\gamma)$ (Blandford et al. 1991), which describes the correlations in the ellipticities of galaxy images as a function of angular separation γ. The

ellipticity correlation function at zero lag, $C_{pp}(0)$, can be related to the power spectrum using the expression

$$C_{pp}(0) = \frac{16\pi^2 r_g^3}{30} \int_0^\infty P_\phi(k) k^5 dk, \tag{30}$$

where we assumed for simplicity that all the galaxies lie at the same distance r_g (Kaiser 1992; Blandford et al. 1991). From this one sees that

$$\frac{\sigma_{\Delta\theta}}{\Delta\gamma_0} = \frac{\sigma_{\Delta\gamma}}{\sqrt{10}\Delta\gamma_0} = \frac{1}{2} C_{pp}^{1/2}(0)(r_{OL}/r_g)^{3/2}. \tag{31}$$

For most cosmological models and distances, the linear theory prediction of equation (27) gives $\sigma_{\Delta\gamma}/\Delta\gamma_0$ of the order of a few percent. This has to be corrected for the nonlinear effects, which are somewhat uncertain. Theoretical estimates (Kaiser 1992) and N-body simulations (Blandford et al. 1991; Miralda-Escudé 1991) suggest that $C_{pp}(0)$ is unlikely to exceed 10^{-3}. This is also supported by the observational data. Mould et al. (1994) report a detection of a signal with a value $C_{pp}(1' < \gamma < 5') = (5.6 \pm 0.6) \times 10^{-4}$, which, after seeing correction, implies average ellipticity within a few arcminutes radius of about 0.05. Assuming this value we find $\sigma_{\Delta\gamma}/\Delta\gamma_0 \approx 0.08(r_{OL}/r_g)^{3/2}$, with $r_g \approx 0.6 H_0^{-1}$. However, the authors could not exclude the possibility that the observed signal is due to systematic effects. This is also suggested by Fahlman et al. (1994), who report a null detection of average ellipticity within a $2.76'$ radius aperture with a sensitivity of about 1.3%, which after adjustment to r_g above implies an upper limit $\sigma_{\Delta\gamma}/\Delta\gamma_0 < 0.03(r_{OL}/r_g)^{3/2}$. These measurements give average ellipticities in typically arcminute size windows and do not probe $C_{pp}(\gamma)$ on scales below $1'$. Observationally it is difficult to give reliable estimates on smaller scales because one has to distinguish between the signal and the noise from the intrinsic ellipticities of galaxies. A rather weak upper limit on $C_{pp}(0)$ can be obtained simply from the average ellipticity of galaxies, which is of the order of 0.4 and is dominated by intrinsic ellipticities. Despite some uncertainty from the model predictions and observations, it appears unlikely that the relative fluctuations in the image separation angle exceed a level of a few percent.

The conclusion above justifies the assumption that the rms fluctuation is small compared to the measured image separation. The fact that $\sigma_{\Delta\gamma}/\Delta\gamma_0 \ll 1$ also implies that one cannot have multiple images produced by LSS alone. Therefore, multiple images can only be formed from nonlinear structures, such as galaxies or clusters. This conclusion has previously been obtained using N-body simulations by Jaroszynski et al. (1991) and using semi-analytical methods by Bartelmann & Schneider (1991). The fluctuations in angular image separation, although small, are in most cases larger than the observational errors on the image positions (typically less than 0.01 arcsecond/arcsecond $\sim 10^{-2}$). Therefore,

LSS effects are a major source of uncertainty in the true image positions. Observational efforts in trying to determine the image position with precisions below 0.01 arcsecond are redundant and do not improve the lens reconstruction. This effect fundamentally limits our ability to reconstruct the lens potential using the image positions and should be included in the modelling of lens parameters, once the actual amplitude of fluctuations is determined from the ellipticity correlation function measurements.

We should also justify the assumptions made in our calculations. One is that the statistical properties of the potential along the perturbed path are well approximated by those along the unperturbed path, as expressed in equation (17). Taylor expansion of the potential gives

$$\phi(\boldsymbol{x}) \approx \phi(r\hat{\boldsymbol{r}}) + \boldsymbol{x}_{\perp} \cdot \boldsymbol{\nabla}_{\perp}\phi(r\hat{\boldsymbol{r}}). \tag{32}$$

Inserting this into the left-hand side of equation (17), we find that for Gaussian random fields the relative correction to the right hand side of equation (17) is given approximately by $(x_{\perp}k_0)^2 \approx [3Q_2(k_0 r_{OL})^{3/2}]^2 \approx 10^{-3}(k_0 r_{OL}/100)^3$. Therefore, the approximation in equation (17) leads to negligible errors. Another approximation we used was to neglect the correlations between LSS and the primary lens. This is justified because the two are correlated only over a correlation length distance, which is much smaller than the typical pathlength. While there are N uncorrelated regions of size k_0^{-1} along the photon path, only one of those is strongly correlated with the primary lens. The contribution from that region can be regarded as being part of the primary lens itself. The error due to this approximation is therefore of the order of $N^{-1} = (k_0 r_{OS})^{-1} \ll 1$.

4. Fluctuations in time delay

In this section we compute the dispersion in time delay between two images. For this purpose it is useful to define the time delay relative to the normal of the lens plane and not relative to the source-observer line. We define the lens plane at the lens redshift z_l to be orthogonal to what would be the source-observer line in the absence of LSS effects (dotted lines on Figure 2). Relative to the lens plane normal, the incoming and outgoing photon direction vectors in the lens plane are $\boldsymbol{\gamma}^{\boldsymbol{A,in}}$, $\boldsymbol{\gamma}^{\boldsymbol{A,out}}$ and $\boldsymbol{\gamma}^{\boldsymbol{B,in}}$, $\boldsymbol{\gamma}^{\boldsymbol{B,out}}$ for the images A and B, respectively. The difference between the incoming and outgoing photon direction gives the deflection angles in the lens plane, $\hat{\boldsymbol{\alpha}}^A$ and $\hat{\boldsymbol{\alpha}}^B$. These can be obtained by modelling the lens potential using various observational constraints, such as image magnifications, velocity dispersion of the lensing galaxy and/or cluster, positions of other images or arcs, etc.

In the absence of LSS the time delay between two images is given from equation 12 by

$$\begin{aligned}\Delta t &= \frac{1}{2}\left\{r_{LS}[(\gamma_1^{A,in})^2 - (\gamma_1^{B,in})^2] + r_{OL}[(\gamma_1^{A,out})^2 - (\gamma_1^{B,out})^2]\right\} - 2(\psi^A - \psi^B) \\ &= \frac{r_{OL}r_{OS}}{2r_{LS}}[(\gamma_1^{A,in})^2 - ((\gamma_1^{B,in})^2] - 2(\psi^A - \psi^B).\end{aligned} \tag{33}$$

Here ψ^A, ψ^B are the integrals of the primary lens potential for the two rays,

$$\psi = \int_{r_{OL}-\epsilon}^{r_{OL}+\epsilon} \phi(r)dr \tag{34}$$

with $\epsilon/r_{OL} \ll 1$. Equation 33 is the usual time delay expression in the thin lens approximation (e.g. Blandford & Narayan 1986; Blandford & Kochanek 1987; Schneider, Ehlers & Falco 1992). Note that the difference between the two outgoing photon directions gives the observed image splitting.

Adding LSS moves both the source and the observer, for a fixed lens plane (Figure 2). The time delay between the two rays is now given by

$$\begin{aligned}\Delta t &= \frac{1}{2}\int_{r_{OL}}^{r_{OS}} \left\{[\boldsymbol{\gamma}^{A,in} - \boldsymbol{\alpha}_\perp^A(r_{OL}, r)]^2 - [\boldsymbol{\gamma}^{B,in} - \boldsymbol{\alpha}_\perp^B(r_{OL}, r)]^2\right\} dr \\ &+ \frac{1}{2}\int_0^{r_{OL}} \left\{[\boldsymbol{\gamma}^{A,out} + \boldsymbol{\alpha}_\perp^A(r, r_{OL})]^2 - [\boldsymbol{\gamma}^{B,out} + \boldsymbol{\alpha}_\perp^B(r, r_{OL})]^2\right\} dr \\ &- 2\left\{\int_{r_{OL}}^{r_{OS}}[\phi^A(r) - \phi^B(r)]dr + \psi^A - \psi^B + \int_0^{r_{OL}}[\phi^A(r) - \phi^B(r)]dr\right\}.\end{aligned} \tag{35}$$

This equation is similar to equation (12), except that here the geometrical contribution is measured relative to the normal of the lens plane. The first two lines in equation (35) give the geometrical time delay between the lens and the source and between the lens and the observer, respectively. In the third line we have written the gravitational time delay contribution coming from the potential between the lens and the source, from the primary lens potential and from the potential between the lens and the observer, respectively. The LSS ($\boldsymbol{\alpha}_\perp$, ϕ) and primary lens ($\boldsymbol{\gamma}^{in}$, $\boldsymbol{\gamma}^{out}$, ψ) contributions are thus explicitly separated.

Let us calculate the time delay contribution between the ray A and the fiducial ray accumulated between the lens and the observer. The fiducial ray is defined to start perpendicular to the lens plane and end at the observer's positions. The total time delay is obtained by adding a similar contribution from the lens to the source and subtracting the same terms for the ray B. The fiducial ray direction is given by

$$\boldsymbol{\gamma}^{\boldsymbol{f}}(r) = \boldsymbol{\alpha}_\perp(r, r_{OL}) = \int_r^{r_{OL}} \boldsymbol{g}_\perp(r)dr, \tag{36}$$

where $\boldsymbol{g}_\perp(r)$ is computed along the fiducial ray. The direction of the ray A is

$$\boldsymbol{\gamma}^A(r) = \boldsymbol{\gamma}^{\boldsymbol{f}}(r) + \boldsymbol{\gamma}^{A,out} + \Delta\boldsymbol{\alpha}_\perp^A(r, r_{OL}). \tag{37}$$

Here $\Delta\boldsymbol{\alpha}_\perp^A(r, r_{OL})$ is the difference between the LSS induced ray deflections at r and can be calculated using the Taylor expansion of $\boldsymbol{g}_\perp$ around the fiducial ray, as we did in §3. The image ray position relative to the fiducial ray is

$$\boldsymbol{x}_\perp^A(r) = \boldsymbol{x}_\perp^A(r_{OL}) + \boldsymbol{\gamma}^{out,A}(r_{OL} - r) + \int_r^{r_{OL}} \Delta\boldsymbol{\alpha}_\perp^A(r', r_{OL})dr'. \tag{38}$$

The initial lens plane position of the image ray relative to the fiducial ray, $\boldsymbol{x}_\perp^A(r_{OL})$, is not a free parameter, since it has to satisfy the constraint

$$\boldsymbol{x}_\perp^A(0) = 0. \tag{39}$$

From this we obtain

$$\boldsymbol{x}_\perp^A(r) = -\boldsymbol{\gamma}^{out,A} r - \int_0^r \Delta\boldsymbol{\alpha}_\perp^A(r', r_{OL})dr'. \tag{40}$$

The gravitational time delay contribution is obtained from the Taylor expansion of the potential around the fiducial ray,

$$\Delta t_{grav} = \int_0^{r_{OL}} \left\{ \boldsymbol{g}_\perp(r) \cdot \boldsymbol{x}_\perp^A(r) + O[(\boldsymbol{x}_\perp^A(r)^2] \right\} dr. \tag{41}$$

The geometrical time delay contribution is given by

$$\begin{aligned}
\Delta t_{geom} &= \frac{1}{2}\int_0^{r_{OL}} [\boldsymbol{\gamma}^A(r)^2 - \boldsymbol{\gamma}^{\boldsymbol{f}}(r)^2]dr \\
&= \frac{(\boldsymbol{\gamma}^{out,A})^2 r_{OL}}{2} + \int_0^{r_{OL}} \Big[\boldsymbol{\gamma}^{\boldsymbol{f}}(r) \cdot \boldsymbol{\gamma}^{out,A} + \boldsymbol{\gamma}^{\boldsymbol{f}}(r) \cdot \Delta\boldsymbol{\alpha}_\perp^A(r, r_{OL}) \\
&+ \boldsymbol{\gamma}^{out,A} \cdot \Delta\boldsymbol{\alpha}_\perp^A(r, r_{OL}) + \frac{\Delta\boldsymbol{\alpha}_\perp^A(r, r_{OL})^2}{2} \Big] dr.
\end{aligned} \tag{42}$$

Integrating by parts the terms involving $\boldsymbol{\gamma}^{\boldsymbol{f}}(r)$ we find

$$\begin{gathered}
\int_0^{r_{OL}} \boldsymbol{\gamma}^{\boldsymbol{f}}(r) \cdot [\boldsymbol{\gamma}^{out,A} + \Delta\boldsymbol{\alpha}_\perp^A(r, r_{OL})]dr = \\
\int_0^{r_{OL}} \left\{ \boldsymbol{g}_\perp(r) \cdot \int_0^r [\boldsymbol{\gamma}^{out,A} + \Delta\boldsymbol{\alpha}_\perp^A(r', r_{OL})]dr' \right\} dr.
\end{gathered} \tag{43}$$

This is exactly cancelled by the first order term in Δt_{grav} (equation 41). Therefore, $\boldsymbol{\gamma}^{\boldsymbol{f}}$ completely drops out of the time delay expression and using time delay measurements we cannot infer any information on the absolute deflection angle. This is quite remarkable, given that separately the geometrical and gravitational LSS induced fluctuations are approximately 15 yr $(k_0 r_{OS}/100)^{1/2}(r_{OS}/1\text{Gpc})(\Delta\gamma_0/1'')$, much larger than the expected time delay from the primary lens itself, of the order of 0.1 yr $(r_{OS}/1\text{Gpc})(\Delta\gamma_0/1'')^2$.

Adding the geometrical and gravitational time delay contributions we finally obtain

$$\Delta t^A = \frac{(\boldsymbol{\gamma}^{out,A})^2 r_{OL}}{2} + \int_0^{r_{OL}} [\boldsymbol{\gamma}^{out,A} \cdot \Delta\boldsymbol{\alpha}_\perp^A(r) + \frac{1}{2}\Delta\boldsymbol{\alpha}_\perp^A(r)^2 + O((\boldsymbol{x}_\perp^A(r)^2)]dr. \tag{44}$$

The first term in equation (44) gives the largest contribution and is the term that one would also have in the absence of LSS (compare with equation 33). The second term is smaller than the first term approximately by $\Delta\alpha_\perp^A(0, r_{OL})/\boldsymbol{\gamma}^{out} \sim \sigma_{\Delta\gamma}/\Delta\gamma_0$. The last two terms in equation (44) are further suppressed by $\sigma_{\Delta\gamma}/\Delta\gamma_0$ relative to the second term and can be neglected.

What, then, is the LSS induced fluctuation that causes the reconstructed time delay to differ from the true time delay? The observer measures the image separation angle that is almost, but not exactly, given by $\boldsymbol{\gamma}^{A,out} - \boldsymbol{\gamma}^{B,out}$, so that the reconstructed time delay differs somewhat from equation (33). It is not possible to give an exact prediction of the reconstructed time delay without specifying the detailed lens model and taking into account all of the observational constraints. It is clear, however, that the fluctuation in the reconstructed time delay is due only to the fluctuation in the relative angle separation, part of which is described by the second term in equation (44). Given that $\sigma_{\Delta\gamma}/\Delta\gamma_0 \ll 1$, the relative effects on the time delay will also be of that order. We conclude that the time delay fluctuation induced by LSS is of the order of $\sigma_{\Delta\gamma}/\Delta\gamma_0$, which is of the order of a few percent for sources and lenses at cosmological distances.

5. Conclusions

We investigated the LSS effects on measurable properties of gravitational lens systems, in particular on the image positions and time delays. The method we use is based on a geodesic equation in a weakly perturbed flat Robertson-Walker metric and is valid for a general matter distribution between the source and the observer. The advantage of this approach compared to previous work on this subject is that it only assumes the knowledge of evolution of density power spectrum, which can easily be related to other measurements of LSS to obtain quantitative predictions. The same approach can also be used to calculate light propagation in non-flat universes and the conclusions in this paper do not significantly depend on the assumed value of Ω. We find that the rms fluctuation in the relative positions of images is $\sigma_{\Delta\gamma}/\Delta\gamma_0 \sim 0.025(k_0 r_{OL}/100)^{3/2}$, for a LSS density power spectrum peaking at wavelength k_0. For most realistic models of LSS this is much smaller than unity and so one does not expect multiple images generated from LSS. Nevertheless, these fluctuations are

likely to be larger than the observational errors and should be included in the modelling of lens parameters. Similarly, rms fluctuation in time delay due to LSS are caused only by the uncertainties in the relative image positions and are also approximately given by $0.025(k_0r_0/100)^{3/2}$. Therefore, LSS does not significantly affect the time delays. While the same method can be used to predict fluctuations in the relative image magnification and orientation, a simple estimate shows that the effect on these observables is negligible. The rms fluctuation in relative magnification between two images $\Delta M/M$ is given approximately by $\Delta M/M \sim (\nabla_\perp M)r_{OS}\Delta\gamma_0/M \sim (k_0r_{OS}\Delta\gamma_0)C_{pp}(0)^{1/2} \sim 10^{-5}$, well below the measurement errors.

The conclusion regarding LSS effects on time delay measurements disagrees with the conclusions reached by Alcock & Anderson (1985, 1986), Watanabe et al. (1992) and Sasaki (1993). Sasaki (1993) suggests that the coupling between the absolute and relative deflection, which in our language is proportional to $\sigma_\gamma\sigma_{\Delta\gamma}$, generates large time delay fluctuations. As we have shown in §4, this term actually vanishes once both the gravitational and geometrical time delay contributions are included. Alcock & Anderson (1985, 1986), Watanabe et al. (1992) and Sasaki (1993) have argued that although the universe is homogeneous on large scales, individual photons may travel through a region where the density differs from the average density in the universe. If one assumes that the density in the beam is some constant fraction of the average density, then one can use the Dyer-Roeder angular distance redshift relation (see Sasaki 1993 for a comprehensive discussion) to deduce the value of the Hubble constant. The result depends on the unknown density in the beam and one may conclude that this significantly affects the Hubble constant determination from the time delay measurements. For example, Alcock & Anderson (1985, 1986) argue that because of LSS correlations there may be a significant overdensity in the particular direction of the lens. This conclusion neglects the fact that LSS and the primary lens are correlated over only a correlation length distance, which is typically much smaller than the photon travel distance. Similarly, Watanabe et al. (1992) and Sasaki (1993) argue that we might live in a universe where part or all of the mass is concentrated in small clumps away from the photon beam, which are unable to significantly affect the light propagation. Both descriptions of LSS are oversimplified in that they do not account for the stochastic nature of LSS. Dynamical measurements over the last decade show large amounts of dark matter present on all scales and so one cannot assume that photons are travelling through a uniformly filled or empty beam when away from the primary lens. Moreover, the average redshift-angular distance relation should not differ from the homogeneous case, once we average over all lines of sight (including those through the clumps of matter). If redshift-angular distance relation for most lines of sight is to differ significantly from that in the homogeneous universe, then this requires large rms fluctuation around the

mean. This would imply large σ_γ/γ_0 but, as we argued in §3, no such large fluctuations are either expected or observed. Therefore, the use of angular distances as given by the homogeneous universe model appears to be adequate. Another reason for the disagreement with Watanabe et al. (1992) is that these authors assumed a large contribution to the fluctuations from small (galactic) scales. As we argued, the data at present show little support for such a large contribution on those scales, although for a more quantitative prediction better measurements of power spectrum on small (subarcminute) scales would be needed.

If LSS does not induce significant time delay fluctuations, then this would remove one of the major objections against using time delay measurements to determine H_0. Significant problems related to the robustness of the lens reconstructions still remain and are preventing the method at present from giving a reliable estimate of H_0 (see e.g. Bernstein, Tyson & Kochanek 1993 for a discussion of lens reconstruction in 0957+561). Moreover, the above analysis does not exclude the possibility that a homogeneous sheet of matter is present in the lens plane, because in our treatment this sheet of matter is part of the primary lens. Uniform matter distribution, which is likely to be overdense close to the primary lens, cannot be determined from the image positions, but it does affect the length scale and makes the deduced value of Hubble constant larger than the true value (Borgeest & Refsdal 1984; Falco, Gorenstein & Shapiro 1991). Although severe, these problems are not unsolvable and GL time delay method remains one of the few methods that can provide information on the global distance scale and geometry of the universe.

I would like to thank Ed Bertschinger for a careful reading of the manuscript and useful discussions. I would also like to thank Richard Gott, Jim Gunn, Lam Hui, Nick Kaiser, Jordi Miralda-Escudé, Ramesh Narayan, Jerry Ostriker, Misao Sasaki, Ed Turner, Kazuya Watanabe and the participants of the seminar on gravitational lenses at MIT for useful discussions. This work was supported by grants NSF AST90-01762 and NASA NAGW-2807.

REFERENCES

Alcock, C., & Anderson, N. 1985, ApJ, 291, L29

Alcock, C., & Anderson, N. 1986, ApJ, 302, 43

Babul, A., & Lee, M. H. 1991, MNRAS, 250, 407

Bardeen, J. 1980, Phys. Rev. D, 22, 1882

Bartelmann, M. & Schneider, P. 1991, A&A, 248, 349

Bernstein, G. M., Tyson, J. A., & Kochanek, C. S. 1993, AJ, 105, 816

E. Bertschinger in: New Insights into the Universe, ed. V. J. Martinez, M. Portilla and D. Saez (Springer-Verlag, New York).

Bertschinger, E. 1993, Les Houches lecture notes

Blandford, R. D., & Narayan, R. 1986, ApJ, 310, 568

Blandford, R. D., & Kochanek, C. S. 1987, in: Dark Matter in the Universe, eds. Bahcall, J., Piran, T., & Weinberg, S. (World Scientific: Singapore)

Blandford, R. D., Saust, A. B., Brainerd, T. G., & Villumsen, J. V. 1991, MNRAS, 251, 600

Bond, J.R., & Efsthathiou, G. 1987, MNRAS, 226, 665

Borgeest, U., & Refsdal, S. 1984, A&A, 141, 318

Cayón, L., Martínez-González, E., & Sanz, J. L. 1993, ApJ, 403, 471

Cayón, L., Martínez-González, E., & Sanz, J. L. 1993, ApJ, 413, 10

Falco, E. E., Gorenstein, M. V., & Shapiro, I. I. 1991, ApJ, 372, 364

Heath, D. J., 1977, MNRAS, 179, 351

Jaroszynski, M., Park, C., Paczynski, B., & Gott, J., R., III 1991, ApJ, 365, 22

Kaiser, N. 1992, ApJ, 388, 272

Fahlman, G. G., Kaiser, N., Squires, G. & Woods, D. 1994, CITA preprint

Kofman, L., & Starobinsky, A. 1985, Sov. Astron. Lett., 11, 271

Lehár, J., Hewitt, J. N., Roberts, D. H., & Burke, B. F. 1992, ApJ, 384, 453

Linder, V. E. 1990, MNRAS, 243, 362

Miralda-Escudé, J. 1991, ApJ, 380, 1

Mould, J., Blandford, R., Villumsen, J., Brainerd, T., Smail, I., Small, T., & Kells, W. 1994, Caltech preprint

Mukhanov, V. F., Feldman, H. A., & Brandenberger, R. 1992, Phys. Rep. 215, 206

Nityananda, R., & Ostriker, J. P. 1984, J. Ap. Astr., 5, 235

Refsdal, S. 1964, MNRAS, 128, 307

Refsdal, S. 1966, MNRAS, 132, 101

Rhee, G. 1991, Nature, 350, 211

Roberts, D., H., Lèhar, J., Hewitt, J. N., & Burke, B., F. 1991, Nature, 352, 43

Sachs, R. K., & Wolfe, A. M. 1967, ApJ, 147, 73

Sasaki, M. 1993, Prog. Theor. Phys. 90, 753

Schneider, P., Ehlers, J., & Falco, E. E. 1992, Gravitational Lenses (Springer-Verlag: New York)

Seljak, U., & Bertschinger, E. 1993, ApJ, 417, L9

Smoot, G. F. et al. 1992, ApJ, 396, L1

Vanderriest, C., Schneider, J., Herpe, G., Chevreton, M., Moles, M., & Wlerick, G. 1989, A&A, 215, 1

Watanabe, K., Sasaki, M., & Tomita, K. 1992, ApJ, 394, 38

Weinberg, S. 1972, Gravitation and Cosmology (Wiley: New York)

This preprint was prepared with the AAS LaTeX macros v3.0.

Fig. 1.— Photon propagation relative to the source-observer line.

Fig. 2.— Schematic diagram of a typical lensing case, as discussed in the text. Solid lines represent true photon trajectories, dashed lines apparent trajectories as seen from the observer's position and dotted lines the unperturbed trajectories as seen from the lens plane in the absence of LSS effects. The apparent image and lens positions are denoted by A', B' and L', respectively, and can be far from the true positions.